APPLICATIONS OF FUZZY GAME THEORY USING EPSILON FUZZY PAYOFF MATRIX

M.Suresh [#1], A.Sathishkumar [*2], S.Deepa [**3]

[#] *Department of Mathematics,*
Dr.R.A.N.M College of Arts and Science, Erode, Tamilnadu, India
[1] msuresh201@gmail.com

[*] *Department of Mathematics,*
Nandha Arts & Science College, Erode, Tamilnadu, India
[2] sathishattssa@gmail.com

[**] *Department of Mathematics,*
Bharathidasan College of Arts and Science, Erode, Tamilnadu, India
[3] sdeepamath@gmail.com

Abstract— **The objective of this paper to construct the two-person zero sum game with Epsilon-delta fuzzy payoff matrix, Triangular epsilon fuzzy payoff matrix and its application by using max-min approximation, fuzzy optimum strategy is evolved.**

Keywords: Fuzzy two person zero sum game, Epsilon fuzzy numbers and fuzzy optimal strategies.

I. INTRODUCTION

The set under consideration is said to be fuzzy set. It is denoted by $\mu_A(x) : X \to [0,1]$ or A : $X \to [0,1]$. Each fuzzy set is completely and uniquely defined by one particular $membership\ function$. Given an fuzzy set A defined on X. Then any number of α − cut can be associated with A is defined by $A_\alpha = \{x \,/\, \mu_A(x) \geq \alpha\}$. Given an fuzzy set A defined on X. Then any number of $strong\ \alpha - cut$ can be associated with A is defined by $A_{\alpha^+} = \{x \,/\, \mu_A(x) > \alpha\,\}$[8]. A convex and normalized fuzzy in R whose membership functions is piecewise continuous is called $fuzzy\ number$. A fuzzy number X is a subset of real line R with membership function X : R → [0,1] such that a fuzzy number X is normal and fuzzy convex and α-cut is closed for all $\alpha \in [0,1]$. Suppose a fuzzy number X is bounded and if the left hand curve and right hand curve are straight lines then the fuzzy number is called the $triangular\ fuzzy\ number$[5]. If r is a real number then ε-fuzzy number is denoted by r_ε is also called $symmetric\ epsilon\ fuzzy\ number$ is the triangular fuzzy number for some $\varepsilon \in$ X ,($\varepsilon >$ 0) is a fuzzy set $r_\varepsilon : X \to [0,1]$ is denoted by,

$$r_\varepsilon(x) = \begin{cases} \frac{x-(r-\varepsilon)}{\varepsilon} & if\ \ r-\varepsilon < x \leq r \\ \frac{x-(r+\varepsilon)}{\varepsilon} & if\ \ r < x \leq r+\varepsilon \\ 0 & otherwise \end{cases}$$

A triangular fuzzy number X = (l, m, n) in above notation is denoted by A = $m_{m-l,n-m}$. Also $r_\varepsilon = (r-\varepsilon, r, r+\varepsilon)$. A ε- δ Fuzzy number $r_{\varepsilon,\delta}$ is $(r-\varepsilon(1-\alpha), r+\delta(1-\alpha))$, r ∈ R, $\varepsilon, \delta \in$X and $\varepsilon, \delta > 0$, then $\propto -\, cut\ of\ \varepsilon - \delta\ fuzzy\ number$ is denoted by, $(r_{\varepsilon,\delta})_\alpha$=[r - $\varepsilon(1-\alpha)$, r +$\delta(1-\alpha)$], $\alpha \in [0,1]$

The following evaluations are $triangular\ approximations\ of\ max\ and\ min\ operations$ of fuzzy numbers,

(i) If r_{ϵ_1,δ_1} and s_{ϵ_1,δ_1} be any two epsilon-delta fuzzy numbers where r ≤ s and r-$\epsilon_1 \leq$ s-ϵ_2 , r+$\delta_1 \leq$ s+δ_2 then we define ,

$$r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_1 \vee \varepsilon_2, \delta_1 \wedge \delta_2} \text{ and } r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_1 \wedge \varepsilon_2, \delta_1 \vee \delta_2}$$

(ii) If r_{ϵ_1,δ_1} and s_{ϵ_1,δ_1} be any two epsilon-delta fuzzy numbers where r ≤ s and s-$\epsilon_2 < r - \epsilon_1$, r+$\delta_1 \leq$ s+δ_2 then we define ,

$$r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_2-(s-r),\delta_1} \text{ and } r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_1+(s-r),\delta_2}$$

(iii) If r_{ϵ_1,δ_1} and s_{ϵ_1,δ_1} be any two epsilon-delta fuzzy numbers where r $\leq s$ and r-$\epsilon_1 \leq$ s-ϵ_2s+$\delta_2 <$ r+δ_1 then we define ,

$r_{\varepsilon_{1,\delta_1}} \wedge s_{\varepsilon_{2,\delta_2}} = r_{\varepsilon_{1,\delta_2+(s-r)}}$ and $r_{\varepsilon_{1,\delta_1}} \vee s_{\varepsilon_{2,\delta_2}} = s_{\varepsilon_2,\delta_1-(s-r)} \vee$

(iv) If r_{ϵ_1,δ_1} and s_{ϵ_1,δ_1} be any two epsilon-delta fuzzy numbers where r $\leq s$ and s-$\epsilon_2 < r - \epsilon_1$, s+$\delta_2 < r + \delta_1$ then we define ,

$r_{\varepsilon_{1,\delta_1}} \wedge s_{\varepsilon_{2,\delta_2}} = r_{\varepsilon_2-(s-r),\delta_2+(s-r)}$ and $r_{\varepsilon_{1,\delta_1}} \vee s_{\varepsilon_{2,\delta_2}} = s_{\varepsilon_1+(s-r),\delta_1-(s-r)}$

Similarly, we get the results for the triangular epsilon fuzzy number by setting $\epsilon = \delta$. The sets of the possible feasible strategies of player-I are two fuzzy sets X and Y on R_1 and R_2 respectively. The two payoff functions P_1 (for player-I) and P_2 (for player-II) from $R_1 \times R_2 \rightarrow [0,1]$ are called *fuzzy two person zero sum game*[1]. An accentual measure of gratification of a person expressed in terms of epsilon fuzzy numbers is called the *epsilon payoff matrix*[2] . For player X has m activities and player Y has n activities. Then the *fuzzy payoff matrix* can be formed by some following rules. The player X's fuzzy payoff matrix is,

	Player Y						
Player X	Strategies	1	2	…..	j	…..	n
	1	$r^{11}_{\varepsilon_{11},\varepsilon_{11}}$	$r^{12}_{\varepsilon_{12},\varepsilon_{12}}$	…..	$r^{1j}_{\varepsilon_{1j},\varepsilon_{1j}}$	…..	$r^{1n}_{\varepsilon_{1n},\varepsilon_{1n}}$
	2	$r^{21}_{\varepsilon_{21},\varepsilon_{21}}$	$r^{22}_{\varepsilon_{22},\varepsilon_{22}}$	…..	$r^{2j}_{\varepsilon_{2j},\varepsilon_{2j}}$	…..	$r^{2n}_{\varepsilon_{2n},\varepsilon_{2n}}$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	i	$r^{i1}_{\varepsilon_{i1},\varepsilon_{i1}}$	$r^{i2}_{\varepsilon_{i2},\varepsilon_{i2}}$	…..	$r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}$	…..	$r^{in}_{\varepsilon_{in},\varepsilon_{in}}$
	⋮	⋮	⋮	…..	⋮	…..	⋮
	m	$r^{m1}_{\varepsilon_{m1},\varepsilon_{m1}}$	$r^{m2}_{\varepsilon_{m2},\varepsilon_{m2}}$	…..	$r^{mj}_{\varepsilon_{mj},\varepsilon_{mj}}$	…..	$r^{mn}_{\varepsilon_{mn},\varepsilon_{mn}}$

The above payoff matrix we denoted by,

$$X_{EP,EP} = \begin{bmatrix} r^{11}_{\varepsilon_{11},\varepsilon_{11}} & r^{12}_{\varepsilon_{12},\varepsilon_{12}} & \cdots\cdots & r^{1n}_{\varepsilon_{1n},\varepsilon_{1n}} \\ r^{21}_{\varepsilon_{21},\varepsilon_{21}} & r^{22}_{\varepsilon_{22},\varepsilon_{22}} & \cdots\cdots & r^{2n}_{\varepsilon_{2n},\varepsilon_{2n}} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ r^{m1}_{\varepsilon_{m1},\varepsilon_{m1}} & r^{m2}_{\varepsilon_{m2},\varepsilon_{m2}} & \cdots\cdots & r^{mn}_{\varepsilon_{mn},\varepsilon_{mn}} \end{bmatrix}_{m\times n}$$

Where,

$$[A] = \begin{bmatrix} r^{11} & r^{12} & \cdots\cdots & r^{1n} \\ r^{21} & r^{22} & \cdots\cdots & r^{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ r^{m1} & r^{m2} & \cdots\cdots & r^{mn} \end{bmatrix}_{m\times n},$$

$$EP = [\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots\cdots & \varepsilon_{1n} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots\cdots & \varepsilon_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \varepsilon_{m1} & \varepsilon_{m2} & \cdots\cdots & \varepsilon_{mn} \end{bmatrix}_{m\times n}$$

and [3] put $\left(r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}\right)$ be the m×n payoff matrix for a two person zero sum game. If $\underline{r}$ denotes the maximin value and $\overline{r}$ be the minimax value of the game then $\overline{r} \geq \underline{r}$.

i,e,

$$\bigwedge_{1\leq j\leq n}\left[\bigvee_{1\leq i\leq m}\{r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}\}\right] \geq \bigvee_{1\leq i\leq m}\left[\bigwedge_{1\leq j\leq n}\{r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}\}\right]$$

If $\bigwedge_{1\leq j\leq n}\left[\bigvee_{1\leq i\leq m}\{r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}\}\right] = \bigvee_{1\leq i\leq m}\left[\bigwedge_{1\leq j\leq n}\{r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}\}\right]$ i.e, $\overline{r} = \underline{r} = r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}$ for all j=1,2,...... n and i=1,2,...... m then the $fuzzy$ game has a $saddle\ point$ at the cell (i,j). If fuzzy payoff $r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}$ is a saddle point then the players have a fuzzy optimal strategies in a pure strategies that is, player-I have ith and player-II have jth fuzzy optimal strategies respectively. The fuzzy payoff $r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}}$ at the saddle point (i,j) is called the value of the game. A fuzzy game is said to be fair if the saddle point of the fuzzy game is zero. That is $r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}} = 0_{\varepsilon,\varepsilon}$.

II. PROBLEM DEFINITION:

In our agriculture country like India there are direct and indirect opportunities in largest economic scale. The present study has been made in Namakkal district of Tamilnadu state. The primary data obtained from the decision maker is not deterministic. Hence it is necessary to process this non deterministic data by fuzzy set theory. Sugar industry needs to attract more sugarcane producers in order to optimize the benefit against various constraints such as near distance, maximum rate, more reliability, good recovery, less deduction, more availability of sugarcane in own zone etc. In present problem we develop payoff matrix by considering only four constraints.

Development of Fuzzy Decision Making Model :

Let A and B are two different sugar factories in Namakkal district in Tamilnadu state. We consider near distance, maximum rate, more reliability and less deduction as constraints for obtaining optimal strategies.

Primary Data For The Season 2015-2016 :

Factory	Distance (Km)	Reliability	Rate in Rs.	Deduction inRs.
A	4 Km	0.7	Rs.2500	Rs.150
B	13 Km	0.8	Rs.2700	Rs.108

Normalization of The Data :

Factory	Distance	Reliability	Rate in	Deduction
A	0.24	0.47	0.48	0.58
B	0.76	0.53	0.52	0.42

We define payoff matrix considering the following rules,

To find diagonal elements:

$P_{ii} = B_i - A_i$, i = 1,2,3,4.

To find non diagonal elements: $P_{ij} = A_i \times B_j$ i≠ j

Calculations:

$P_{11} = B_1 - A_1 =$ 0.76-0.24= 0.52, $P_{12} = A_1 \times B_2 =$ 0.24× 0.53= 0.13. In this way, we can find the following payoff matrix,

Factory B

Factory A

	Near Distance	More Reliability	Maximum Rate	Less Deduction
Near Distance	0.52	0.13	0.12	0.10
More Reliability	0.36	-0.06	-0.24	-0.20
Maximum Rate	0.36	-0.25	-0.04	-0.20
Less Deduction	0.44	0.31	0.30	-0.16

Fuzzy payoff matrix :

Factory B

Factory A		Near Distance	More Reliability	Maximum Rate	Less Deduction
	Near Distance	$52_{10,10}$	$13_{10,10}$	$12_{10,10}$	$10_{10,10}$
	More Reliability	$36_{10,10}$	$-6_{10,10}$	$-24_{10,10}$	$-20_{10,10}$
	Maximum Rate	$36_{10,10}$	$-25_{10,10}$	$-4_{10,10}$	$-20_{10,10}$
	Less Deduction	$44_{10,10}$	$31_{10,10}$	$30_{10,10}$	$-16_{10,10}$

MAX-MIN Approximation:

ROW MINIMUM:

Row I: $52_{10,10} \wedge 13_{10,10} \wedge 12_{10,10} \wedge 10_{10,10} = 10_{10,10}$

ROW II: $36_{10,10} \wedge -6_{10,10} \wedge -24_{10,10} \wedge -20_{10,10} = -24_{10,10}$
Row III: $36_{10,10} \wedge -25_{10,10} \wedge -4_{10,10} \wedge -20_{10,10} = -25_{10,10}$
Row IV: $44_{10,10} \wedge 31_{10,10} \wedge 30_{10,10} \wedge -16_{10,10} = -16_{10,10}$

Fuzzy Maxi-Min: $10_{10,10} \vee -24_{10,10} \vee -25_{10,10} \vee -16_{10,10} = 10_{10,10}$

COLUMN MAXIMUM:

Column I : $52_{10,10} \vee 36_{10,10} \vee 36_{10,10} \vee 44_{10,10} = 52_{10,10}$

Column II : $13_{10,10} \vee -6_{10,10} \vee -25_{10,10} \vee 31_{10,10} = 31_{10,10}$

Column III: $12_{10,10} \vee -24_{10,10} \vee -4_{10,10} \vee 30_{10,10} = 30_{10,10}$

Column IV: $10_{10,10} \vee -20_{10,10} \vee -20_{10,10} \vee -16_{10,10} = 10_{10,10}$

Fuzzy Minimax: $52_{10,10} \wedge 31_{10,10} \wedge 30_{10,10} \wedge 10_{10,10} = 10_{10,10}$

Thus, $\wedge (\vee r^{ij}_{\varepsilon_{ij},\delta_{ij}}) = 10_{10,10} = \vee (\wedge r^{ij}_{\varepsilon_{ij},\delta_{ij}})$

Thus, the fuzzy optimal strategy for factory A is I (Near distance) and the fuzzy optimal strategy for factory B is IV (Less deduction).

III. CONCLUSION

Thus we consider a fuzzy two person zero-sum game with epsilon fuzzy numbers and also this paper focuses the development of the applications of fuzzy game theory to industrial decision making. The information given by farmers in non-deterministic and is modeled in terms of fuzzy sets The 'fuzzy minimax-maximin criterion' is used for obtaining best optimal strategy for sugar factory A and B.

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